Reply to "Comment on 'Andronov bifurcation and excitability in semiconductor lasers with optical feedback' "

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We reply to the comment on our paper [Phys. Rev. E **55**, 6414 (1997)]. We discuss each one of the arguments issued in the comment about our interpretation of the experimental results: the role of noise, the excitable character of the system, and the existence of an Andronov bifurcation. We clarify what the purpose of our analysis was and we discuss its relation with the existing theory. [S1063-651X(98)00709-0]

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van Tartwijk and Fisher [1] discuss the main claims of Ref. [2]: the role of noise, the excitable character of the system, and the existence of an Andronov bifurcation. Moreover, they affirm the validity of a commonly used theoretical model for a semiconductor laser with optical feedback in order to explain the experimental results reported in [2]. First we would like to clarify how we interpret our experimental results. Then we will discuss the theoretical problem.

In Ref. [2] we stated explicitly that our interest is to identify the type of bifurcation leading to low-frequency fluctuation (LFF) instability. This question is meaningful only if the system is deterministically and not stochastically driven; then a bifurcation may be observed and characterized. Independently of the existence of the "fast" pulses (i.e., pulses with characteristic frequencies greater than the experimental bandwidth) described in [1], a time averaged variable will decrease the relevance of noise. Moreover, if a bifurcation is observed in the time averaged variable, then it has to be present in the "real time" signal. Restricting our analysis to the transition between a stable averaged intensity output and the so-called LFF regime, we showed that a return map shows a cloud of points [Fig. 7(a) of Ref. [2]].

If we assume that the system is deterministically driven, this would require a bifurcation with codimension greater than 2 in order to justify a cloud of points in the return map. Our experimental result is general for a very wide range of values of feedback strength [from a level involving 8% threshold reduction with respect to the solitary laser up to the maximum level available in our setup (20%)] and external cavity length (0.1–0.5 m); then only noise induced effects can explain such return map.

The averaging of the temporal behavior of the intensity around the dropout [Fig. 5(a) of Ref. [2]] is again a more restrictive test than observing the time resolved intensity. In fact, the process of averaging tends to increase correlation lengths that may be washed out by fast pulsing. We showed that there is loss of correlation during the intervals between the drops of the intensity when the system operates closed to the critical parameter value at which LFFs appear.

The two above results, together with the histograms of Fig. 6(a) in [2], give the only possible conclusion: Noise plays a relevant role close to the bifurcation point. However, we stated clearly that a bifurcation exists somewhere in parameter space and that the deterministic behavior takes over the noise at higher pump rates, as shown in Figs. 5(b), 6(b), and 7(b) of Ref. [2]. We have never explored the relative size of the region in parameter space for which noise plays the dominant role and Refs. [11,12] cited in Ref. [1] are still not available to us.

In conclusion, van Tartwijk and Fisher [1] misunderstood the sense of our discussion about the role of noise. Our measurements tried only to justify the existence of determinism and therefore the validity of the question we put to ourselves: Which type of bifurcation is at the origin of the power drops? In order to identify such bifurcation we performed a series of tests. The fluctuations of the intensity observed in our system have two main characteristics: low frequency and strong amplitude. Such a type of instability can be generated by two processes: a subcritical Hopf bifurcation or an Andronov bifurcation. The first one is characterized by the existence of multistability between the fixed point and a limit cycle and, in general, the frequency increases approaching the bifurcation point. The latter one is a global bifurcation usually produced after the collision of a fixed point with a saddle giving rise, in phase space, to an orbit that remains for a long time in the neighborhood of the preexisting fixed point and it evolves fast far away from it. Such a bifurcation is characteristic, for example, of a pendulum in a rotating wind or a laser with injected signal [3] and it is easily described by Adler's equation. The orbit generated after the bifurcation may evolve close to other unstable steady states before returning to the position of the fixed point and the saddle that were at its origin. The experimental observation of the frequency decreasing as we approach the bifurcation point, the absence of bistability between a fixed point and a limit cycle, and the already mentioned low frequency and high amplitude of the oscillations lead to the only possible interpretation: an Andronov bifurcation. Contrary to what is written in Ref. [1], an Andronov bifurcation is not simply the collision of a fixed point with a saddle (usually called a saddle-node bifurcation). An Andronov bifurcation is a global bifurcation involving a connection between the stable manifold of the attractor with the unstable manifold of the saddle [4]. van

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Tartwijk and Fisher [1] suggest an interpretation based on a process called chaotic itinerancy with a drift. To our knowledge, chaotic itinerancy with a drift does not exist as a type of bifurcation in nonlinear dynamics. It is important to note that Ref. [5] does not provide evidence of such a process that involves a time scale equal to the time between intensity drops. The experimental measurements shown in [5] cover, at most, one-fifth of such time, thus it is completely useless in order to make a comparison with a process involving much longer time scales. In fact, we recently demonstrated [6] that the dynamics of the LFF is associated intrinsically with the multimode operation of the laser. Thus the low-frequency fluctuations involve many fixed points. We also stated clearly in Ref. [2] that an Andronov bifurcation is usually associated with an excitable character of the system [7]. van Tartwijk and Fisher [1] criticize our conclusion because ''the amplitude of the perturbation applied is larger than the LFF regime.'' It was clearly explained in [2] that excitability is not recognized by the response to a small perturbation. On the contrary, it is recognized by the existence of a critical size of the perturbation above which the response of the system becomes independent of the perturbation itself. Also they [1] stated that we need a perturbation of 10 mA. This is not the case because we showed in Fig. 8(b) of [2] that the critical value for the perturbation is 3 mA. Figure 8(c) of [2] demonstrates that the response of the system was unaltered even if the excitation pulse is as great as 10 mA. So, considering that the LFF regime spans over a region smaller than the perturbation itself and that the response to the excitation remains unchanged, we have very strong experimental evidence of excitability. Moreover, it seems to us that van Tartwijk and Fisher [1] did not understand that the 60 ps width of the excitation induces a change in the initial condition. When the perturbation kicks the system on the ''other side'' of the antimode, the intensity evolves, making a long deterministic trajectory in phase space before returning to its initial state. For this reason the response is independent of the perturbation. If the system realizes that the parameter values change (e.g., for much wider pulses or low-frequency modulations), the response depends strongly on the amplitude of the perturbation.

Finally, Ref. [1] discusses the validity of the Lang-Kobayashi (LK) equations to interpret our results. This is a secondary point in Ref. [2], which was dedicated to the analysis of the type of bifurcation present in the system independently of any theoretical model and to showing excitability in an optical system. However, it must not be a surprise that the LK model is not able to justify many of the observed results in a semiconductor laser with optical feedback from a mirror. The LK model assumes a very low feedback level and single-mode operation of the laser neglecting the spatial dependence of the field inside the medium. Such approximations are clearly not valid for edge emitter semiconductor lasers because the frequency separation among longitudinal modes is much smaller than the gain bandwidth and the strength of the feedback when LFFs appear is two to three orders of magnitude greater than those necessary for LK model to be valid. We have also shown in [6] that the system behaves multimodally when a LFF appears and that statistical measurements of the intensity do not coincide with the ones expected by the LK model in a wide region of parameter space [8].

It is worthwhile to note that van Tartwijk and Fisher [1] claim that the intensity as a function of time measured in Ref. [5] is evidence of the validity of the LK model. Only a bifurcation diagram or the reconstruction of a template can ensure the correspondence (at least topologically) between a model and an experiment. In fact, we could not find in [5] either the qualitative or the quantitative agreement between theory and experiment claimed by van Tartwijk and Fisher [1].

In conclusion, we clarify why our measurements represent experimental evidence of an Andronov bifurcation and excitability in an optical system. We explain what the experimental relevance of performing an assessment of the role played by noise is. We hope that this paper will help van Tartwijk and Fisher [1] and other readers better understand our original paper.

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